Multipole gravity solver for 3D simulations of core-collapse supernovae with CHIMERA

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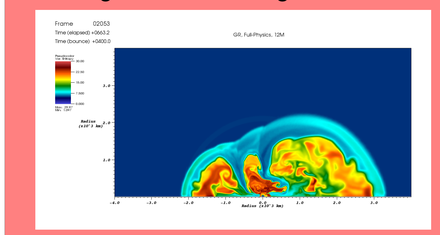
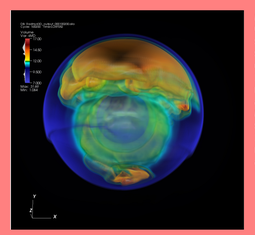
Introduction

The advent of petaflop computing has opened up new possibilities in simulating supernovae. The computing power and sophistication of parallel programming tools allow for a deeper and more complete investigation into the nature and understanding of supernova physics.

CHIMERA is a multi-dimensional numerical code for studying core-collapse supernova. Akin to the mythical creature which is its namesake, CHIMERA is built from several distinct physics modeling codes (Bruenn, 2009).

CHIMERA is capable of running in 1, 2, or 3 spatial dimensions. The 3D code implements the Yin-Yang grid, a new technique in 3D spherical coordinate systems that allows for faster simulations by avoiding coordinate singularities.





Left: A 2D CHIMERA simulation showing the entropy of a 12 solar mass model 400 ms after bounce *Right:* 3D simulation of 15 solar mass model 132 ms after bounce.

The Yin-Yang grid requires a new implementation of the spherical multipole gravity solver.

Spectral Poisson Solver

Introduced by Mueller & Steinmetz (1995), this algorithm efficiently solves Poisson equation for the gravitational potential of self-gravitating hydrodynamic flows in 3D.

The solution to Poisson's equation is given as a multipole expansion of the density distribution.

$$\Phi(r,\theta,\varphi)=-G\sum_{l=0}^{\infty}\frac{4\pi}{2l+1}\sum_{m=-l}^{l}Y_l^m(\theta,\varphi)\left(\frac{1}{r^{l+1}}C_l^m(r)+r^l D_l^m(r)\right)$$

These spherical harmonic expansions are not possible on the Yin-Yang grid.

$$C_l^m(r)=\int_{4\pi}d\Omega'\,Y_l^{m*}(\theta',\varphi')\int_0^r dr'\,r'^{\,l+2}\rho(r',\theta',\varphi')$$

$$D_l^m(r)=\int_{4\pi}d\Omega'\,Y_l^{m*}(\theta',\varphi')\int_r^{\infty}dr'\,r'^{\,1-l}\rho(r',\theta',\varphi')$$

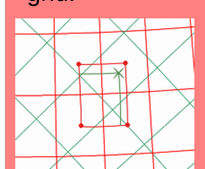


Diagram of the interpolation scheme. Four neighboring values of the original grid are used to calculate the value at the point on the new grid.

To overcome this issue, we implement a full polar auxiliary grid, oriented to be coincident with the Yin grid. The density is then interpolated onto the auxiliary grid from both the Yin and Yang grids using a bilinear interpolation scheme.

The multpole expansion is carried out in a normal fashion on this auxiliary grid. Once the angular weights are determined, we return to the Yin-Yang grids to form the potential and complete the algorithm.

Motivation

Core-collapse supernova are the end result of stars with a mass greater than 10 solar masses and have burned their cores down into iron. Gravitational collapse causes the formation of a neutron star core or black hole, depending on the initial mass. The supernova explosion injects the surrounding interstellar medium with energy and newly formed elements.

An unsolved phenomenon in supernova theory is the mechanism by which the stalled core-collapse supernova shock is revived. Several effects exist that attempt to explain this dilemma, including neutrino-driven convection and SASI (Blondin et al., 2003).

Furthermore, modeling of supernovae allows for insights into the physics of stellar evolution, gravitational waves (Yakunin et al., 2010), and neutrino physics.

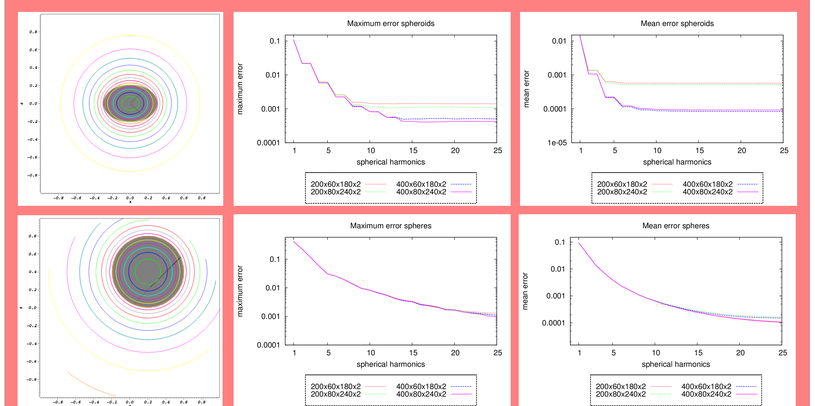
Design

CHIMERA models core-collapse supernova physics using several code components:

- Hydrodynamics (EVH1) (Mezzacappa, 1998)
- Self-gravity: Newtonian + GR extension
- Neutrino transport (MGFLD-TRANS) (Bruenn, 1985)
- Nuclear reaction networks (XNET) (Hix, Thielemann, 1999)

The individual components of CHIMERA are threaded and converted for OpenACC. MPI and OpenMP are utilized, and CUDA development on select components is underway.

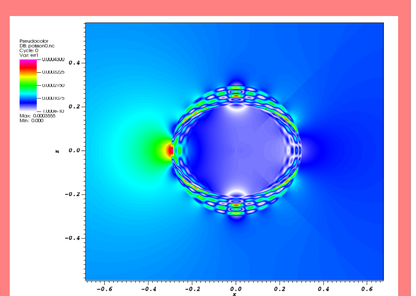
Spectral Poisson Solver



Test cases for the spectral poisson solver using homogeneous density distributions. *Top* spheroid test *bottom* offset sphere test. Plotted *left* is the contour of the gravitational potential *center* is the maximum error of the solver from a known solution and *right* is the mean error of the solver from a known solution.

This modified version of the algorithm has been tested using test cases with known solutions for the gravitational potential.

The algorithm is seen to very good agreement with these tests.

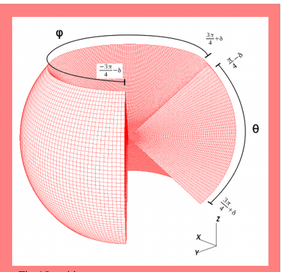


Plot of the relative error in the gravitational potential using a known solution for the spheroid case. Notice the faint Yin-Yang grid boundary.

Yin-Yang Grid

Spherical polar grids suffer from increasingly small zone in the polar regions. This places severe limits on the computational time step.

To alleviate this problem, Kageyama & Sato (2004) introduced the Yin-Yang grid. Two symmetrical grids, which are subsections of a spherical polar grid, are overset to cover the full solid angle of 4π steradians.

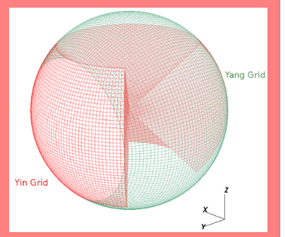


The Yin grid

Each grid is defined in spherical polar coordinates by

$$\frac{\pi}{4}-\delta\leqslant\theta\leqslant\frac{3\pi}{4}+\delta \qquad \frac{-3\pi}{4}-\delta\leqslant\varphi\leqslant\frac{3\pi}{4}+\delta$$

The parameter δ is a small angular width, used to ensure full coverage of the full solid angle.

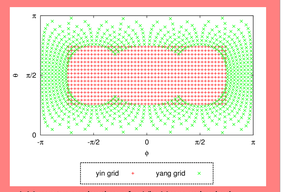


The Yin-Yang grid overset, covering the full solid angle. Notice the overlapping boundary regions, which require special attention.

Between grids, the angular coordinates transform as

$$\theta^e=\arccos(\sin\theta^n\sin\varphi^n) \qquad \varphi^e=\arctan\left(\frac{-\cos\theta^n}{\sin\theta^n\sin\varphi^n}\right)$$

Where the superscripts *n* and *e* denote the Yin and Yang grids, respectively.



A Mercator projection of a Yin-Yang spherical surface.

Future Work

Each code component (hydro, gravity, ect) can run independently during one time step. This feature allows these components to be tasked in parallel, so further improve code efficiency.

References

- Kageyama A., Sato, T. 2004, Geochem. Geophys. Geosyst., 5
- Meuller E., Steinmetz M. 1995 Comp. Phys. Comm. 89 45
- Mezzacappa A., et al. 1998, Astrophysical Journal 495, 911
- Marek A., Dimmelmeier H., Janka H-T., Meuller E., Buras R. 2006, Astron. Astrophys. 445 273
- Blondin J M., Mezzacappa A., DeMarino C. 2003, Astrophysical Journal 584, 971–980
- Bruenn S W. 1985, ApJS 58 771–841
- Bruenn S W. , et al. 2009, J. Phys.: Conf. Ser., 180, 012018
- Hix W. R., Thielemann F. K. 1999, J. Comput. Appl. Math. 109, 321 – 351
- Yakunin K. N., et al. 2010, Classical and Quantum Gravity, 27, 194005. 1005.0779