Massively parallel simulation of anisotropic Debye shielding in the modulator of a coherent electron cooling system and subsequent amplification in a free electron laser

B.T. Schwartz,[1] D.L. Bruhwiler,[1] V.N. Litvinenko,[2] S. Reiche,[3] G.I. Bell,[1] A. Sobol,[1] G. Wang,[2] and Y. Hao[2]

[1] Tech-X Corp., Boulder, CO 80303, USA
[2] Brookhaven National Laboratory, Upton Long Island, New York, USA
[3] Paul Scherrer Institut, Villigen PSI, 5232, Switzerland

Abstract. Novel electron-ion collider concepts are a high priority for the long-term plans of the international nuclear physics community. Orders of magnitude higher luminosity will be required for the relativistic ion beams in such accelerators. Electron cooling is a promising approach to achieve the necessary luminosity. The coherent electron cooling (CeC) concept proposes to combine the best features of electron cooling and stochastic cooling, via free-electron laser (FEL) technology, to cool high-energy hadron beams on orders-of-magnitude shorter time scales. We present δf particle-in-cell simulations of a CeC modulator, using the parallel VORPAL framework. We also present GENESIS simulations relevant to subsequent amplification of the electron wake in a high-gain FEL.

1. Introduction

Increasing the luminosity of hadron accelerators is important for advancing the fields of particle and nuclear physics. One method of achieving this goal is coherent electron cooling (CeC) [4], which involves Debye shielding of moving ions in a plasma with an anisotropic velocity distribution. An analytic solution has been developed for analyzing this case of Debye shielding for a Lorentzian velocity distribution [8]. Here we use this analytic solution to validate particle-in-cell (PIC) simulations of ion shielding in plasmas with isotropic and anisotropic Lorentzian velocity distributions. Having validated the simulation, we apply a more realistic Maxwellian velocity distribution to cases with two closely-spaced ions, showing that the electron response is linear and, hence, additive.

1.1. Coherent Electron Cooling

The primary cooling mechanism in a standard electron cooler is dynamical friction on the ions (see e.g., Ref. [1] and references therein). The modulator section of a coherent cooler would be similar to a standard cooler, but in this case dynamical friction becomes irrelevant and the key physics is the coherent density and velocity perturbations imprinted on the electron distribution via anisotropic Debye shielding of each ion [4].

Coherent electron cooling consists of two components in addition to the modulator: a free-electron laser (FEL) amplifier, and a *kicker* [4]. In the modulator, electron and ion beams copropagate at the same velocity such that the ions imprint a modulation on the electron density. Both beams have highly anisotropic velocity distributions such that $\sigma v_{x,y} \gg \sigma v_z$.

We first consider the motion of a single ion in the beam reference frame. Electrons gather preferentially behind the ion, forming a cloud that shields the ion charge, with density dropping to the background level over several Debye lengths along each dimension. Since the transverse rms velocities exceed the longitudinal, the cloud resembles a pancake for a stationary ion. In the laboratory frame, the longitudinal dimension contracts by the beam's Lorentz factor γ_0, so the pancakes can be very thin.

After the modulator, the electron beam propagates through a high-gain FEL, which amplifies the pancake-shaped electron clouds that had formed in the wake of each ion, resulting in a longitudinal charge density variation with a period equal to the FEL wavelength. Each ion is phase shifted with respect to the electron beam such that copropagation in the kicker leads to an interaction with its own amplified electron wake, decreasing the relative velocity [4].

1.2. Debye Shielding

In isotropic Debye shielding of stationary ions, the steady-state electric potential $\phi \propto \frac{1}{r}$ around the ion decays exponentially as $\exp\left(\frac{-r}{\lambda D}\right)$, where $\lambda_D = \left(\frac{\epsilon_0 \kappa T}{e^2 ne}\right)^{1/2}$ is the Debye length [6].

However, this expression for shielding does not apply to the CeC case for two reasons: (1) the electron-ion interaction time is less than the plasma period, and (2) the electron beam temperature is highly anisotropic. Time-dependent analytical solutions for plasmas with Lorentzian velocity distributions are derived in Ref. [8]. In Sect. 2 we use this solution to validate particle-in-cell simulations of ion shielding in isotropic and anisotropic plasmas with Lorentzian velocity distributions. Having validated the simulation, we then consider a more realistic Maxwellian velocity distribution.

1.3. VORPAL δf PIC Simulations

The ion shielding simulations employ VORPAL's δf PIC algorithm, which uses particle weights to measure the perturbations away from an equilibrium state [9]. The δf PIC method provides significant computational advantages over conventional PIC computations. To obtain even moderate agreement with theory, conventional PIC simulations required volumes of 10^4 Debye cubes, with at least 1,000 particles per cell. These correspond to simulations with approximately 10^7 mesh cells and 10^{10} macroparticles, which took several hours to run on a supercomputer with 2,048 cores. By comparison, the δf PIC simulations shown in Sect. 2 typically required only 200 particles per cell, yielding better agreement and less particle noise, even with lower resolution. Simulations using 2048 cores on the Franklin supercomputer at NERSC typically require 90 minutes to simulate shielding for durations up to half a plasma period.

2. Software Validation

For the case of Debye shielding by electrons with a kappa velocity distribution with $\kappa = 2$ (i.e., a Lorentzian) [2], analytic results have been derived [8]. In this section we validate VORPAL δf PIC simulations for this case by comparing the results to these analytical predictions.

Equilibrium Debye shielding around a single Au^{+79} ion produces an excess of ~79 electrons among ~10^7 physical electrons in the relevant domain. Since the simulation domain is finite, the simulation is set up to allow a thermal flux of electrons to leave and enter the boundaries. Electrons near the boundaries that had just recently entered the simulation domain have errors in their positions and velocities, since they have only begun to experience the fields from the ion and other electrons. Hence, only the central portion of the domain is shown in the Figures 1–4. To obtain good agreement the post-processing did not evaluate the perturbed electron charge within a few Debye lengths (typically 5) of the simulation domain edge.

Figure 1 compares the local integrated shielding charge verses time for the linear analytical prediction and the VORPAL δf PIC simulation. In this case the electron velocity distribution is anisotropic such that $v_x = v_y = 3v_z$ ($R = 3$), where z is the longitudinal direction. The total perturbed

electron charge that accumulates in a finite volume around the ion is expected to peak at half a plasma period, after which the oscillation amplitude dampens, as shown in Figure 2. The plasma with a Maxwellian velocity distribution exhibits oscillatory behavior similar to the Lorentzian case, but with different amplitude and phase.

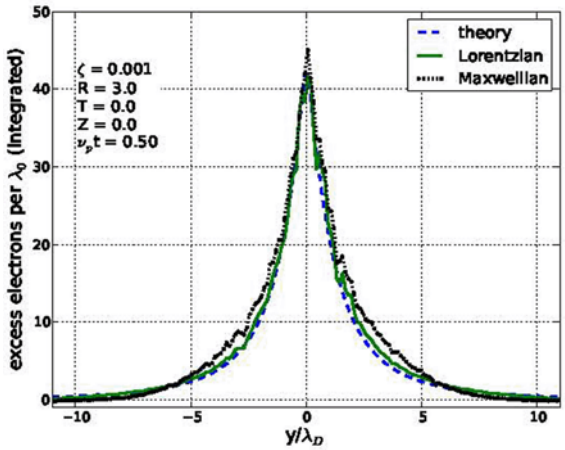

Figure 1. Longitudinal charge density perturbation in the vicinity of the Au^{+79} ion, for the case of a stationary ion in an anisotropic plasma with both Lorentzian and Maxwellian e^- velocity distributions.

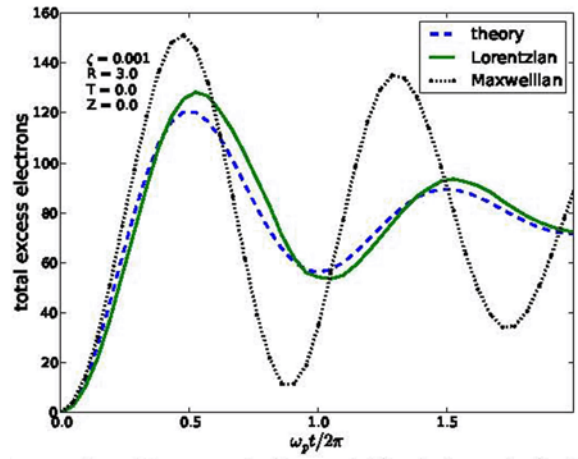

Figure 2. Time evolution of the integrated e^- charge enhancement in the vicinity of the Au^{+79} ion, for the case of a stationary ion in an anisotropic e^- distribution. The time scale is in units of plasma period.

Good agreement between the VORPAL simulations and theory is maintained for moving ions. The simulations in the following plots show Debye shielding for anisotropic plasmas with the ratio of transverse to longitudinal velocity spread of R = 3. Figure 3 and Figure 4 show the shielding charge for a gold ion moving in both the longitudinal (z) and transverse (x) directions. In each case the ion is moving with a transverse velocity of T = 5.4 times the plasma's rms transverse velocity and Z = 0.6 times plasma's rms longitudinal velocity. We note that the Maxwellian and Lorentzian responses can differ significantly, as seen in Figure 4.

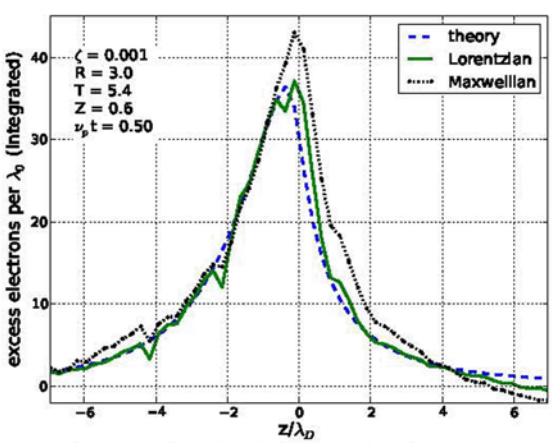

Figure 3. Longitudinal charge density perturbation of a plasma in the vicinity of a moving Au^{+79} ion.

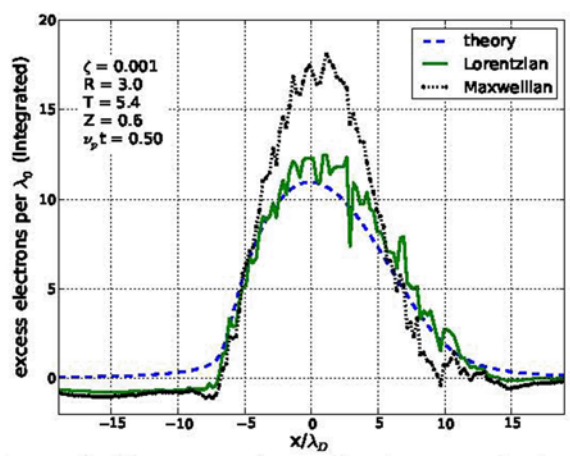

Figure 4. Transverse charge density perturbation of a plasma in the vicinity of a moving Au^{+79} ion.

The following quantities were varied to obtain good agreement of the δf PIC simulations and the theoretical prediction: domain width, number of cells per Debye length, number of particles per cell, and electron velocity range. The domain width ranged from $24\lambda_D$ to $48\lambda_D$, and was sampled with four to eight cells per Debye length. The particles per cell ranged from 200 in most cases, to 800 for the faster moving ions. Modeling such cases also required increasing the electron velocity range from $\pm 5v_{\mathrm{rms}}$ to $\pm 7v_{\mathrm{rms}}$.

3. Debye Shielding of Two Ions

It is important for CeC that the plasma's response to an ion beam be linear, meaning that the total response is approximately the superposition of responses to each ion. Otherwise individual ions will not be properly accelerated or decelerated (cooled) in the kicker. Our initial inquiries into this matter involve electron shielding simulations of two ions separated by different distances. Figure 5 shows a case where plasma has an anisotropic Maxwellian velocity distribution, R = 3. The number of shielding electrons is essentially independent of separation distance; the variation was less than 2%. Hence, the response is linear.

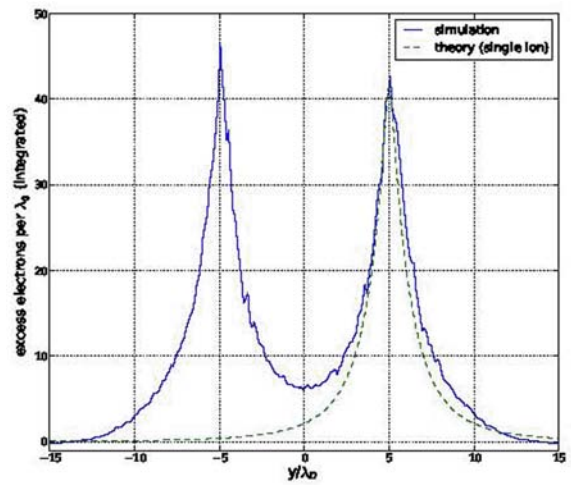

Figure 5. Transverse charge density perturbation of a plasma in the vicinity of two stationary Au^{+79} ions separated by $10\lambda_D$. Dotted line: theoretical prediction for a Lorentzian velocity distribution.

4. Free-Election Laser Amplification of the Electron Wake

After the modulator, the electrons pass through an FEL, which amplifies the electron distribution and modulates it with a period equal to the FEL wavelength. Correct simulation of the coupling from modulator to FEL amplifier involves extracting bunching parameters and associated phases from the electron density histograms shown above in Sect. 2, and using them to properly modify the input electron distribution used by GENESIS [7], an FEL simulation code. We will present the details of such calculations in future work.

As an intermediate step toward converting λf VORPAL outputs to inputs for GENESIS, we used GENESIS to simulate a λ_0 = 700 nm FEL with an electron beam that had a non-zero bunching parameter b only in the leading 1% if its length. This bunching was sufficient to cause lasing via self-amplified spontaneous emission for bunching parameters as small as $b = 10^{-10}$. Figure 6 shows that the FEL imparts a charge density oscillation on the electron beam.

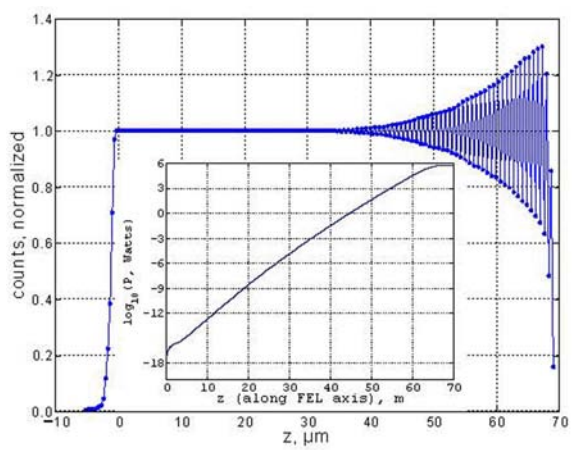

Figure 6. Histogram of electron beam particle counts after propagating 60 meters in an λ_0 = 700 nm FEL, bin width = $\frac{\lambda_0}{2}$. Inset: Optical power along FEL axis.

Using an algorithm based on Refs. [3] and [5], we found the bunching magnitude extracted from this signal at different positions in the FEL to agree fairly well with those computed within GENESIS. To verify correct bunching, we

employed an algorithm identical to the standard definition of bunching and method used in GENESIS: the average the electron ponderomotive phase in each λ_0-wide slice of the electron distribution. Both the bunching magnitude and phases computed using this method agreed very well with the GENESIS calculations.

5. Scaling of λf PIC Simulations on Many Processors

Figure 7 shows the strong scaling of the simulation time for the λf PIC ion shielding simulations shown in Figures 3 and 4. Removing the ion (Z = 0) improved scaling slightly, as the reduced electron motion improved load balancing across MPI domains. Disabling file I/O, notably the output of large files containing charge and current densities, increased efficiency significantly to 92% of the linear scaling value for N = 4096 processors.

Weak scaling efficiencies exceeded 100% for N = 16 to N = 1024 processors compared to an N = 1 reference, with a maximum of 114% for N = 1024.

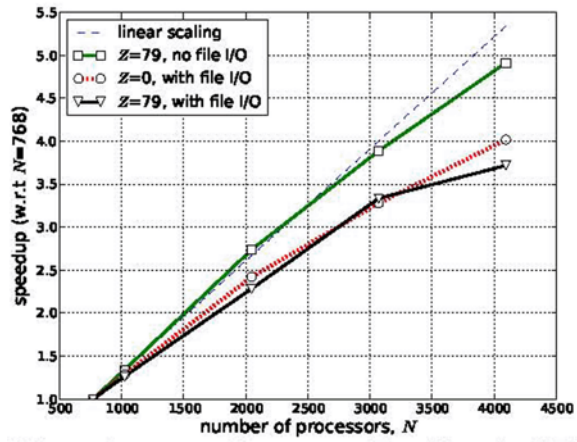

Figure 7. Strong scaling of λf PIC simulations in VORPAL on Franklin, the NERSC Cray XT4 massively parallel processing (MPP) system.

6. Conclusions

VORPAL λf PIC simulations accurately model Debye shielding of both stationary and moving single ions in isotropic and anisotropic plasmas. The shielding response to multiple ions is linear. Since these simulations scale efficiently up to thousands of processors, it will be possible to model shielding of many ions, and hence characterize the modulator section of a CeC system. Initial FEL simulations suggest that the small electron density perturbations associated with Debye shielding can be amplified to create the electric field profiles required reduce relative ion velocities. Parallel computing will enable end-to-end simulations of future CeC experiments.

Acknowledgments

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