Multiscale and Multiphysics Modelling of the Respiratory System

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INRIA

JJL

Young Investigators Symposium

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Oak Ridge National Laboratory

INRIA- Paris 6 University Team

o *Mathematical modeling and Numerical Simulation of Biological Flows,* a joint project-team of INRIA Paris-Rocquencourt and the Jacques-Louis Lions Lab.

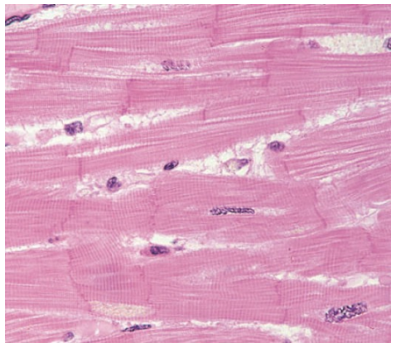
Application field

- Blood flow modeling,
- Electrical activity of the heart,
- Air flow modeling.

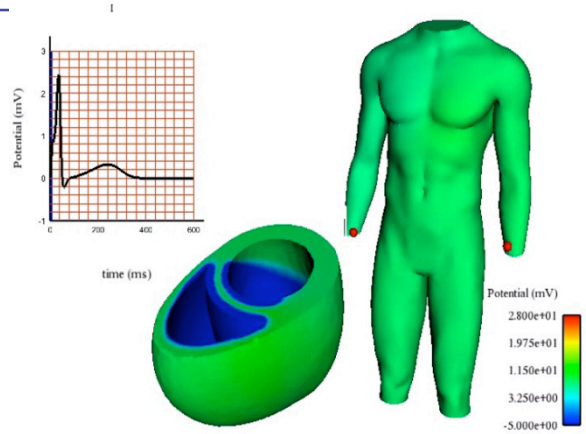
Methodology

- Mathematical modeling,
- Design of efficient schemes,
- Developing simulation software

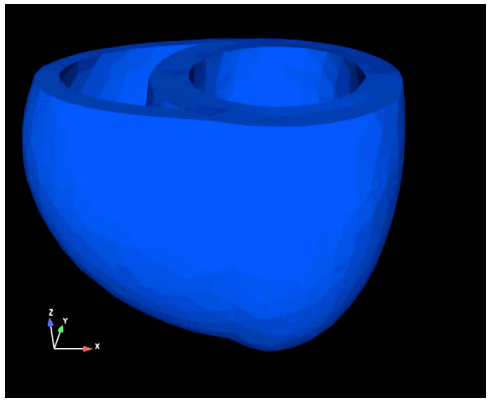
Heart modeling



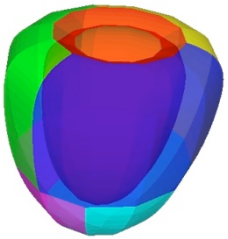
Transport & metabolism



Electrical potential



Excitation contraction coupling



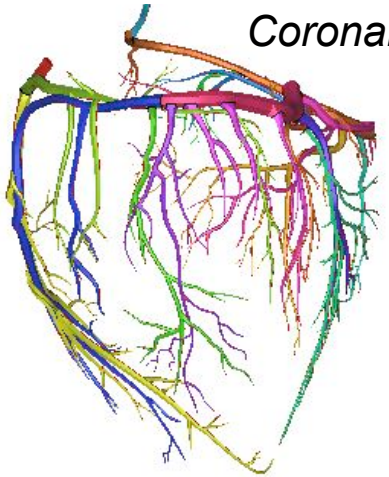
Blood flow

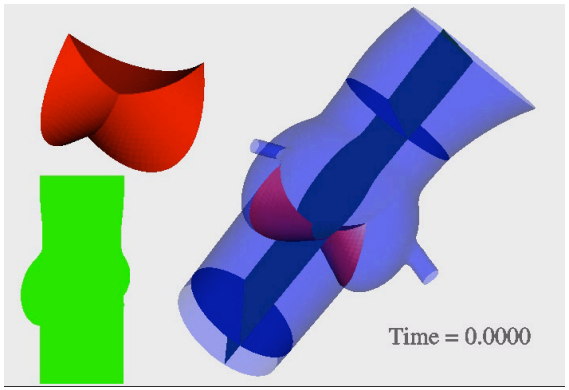
Mechanical stress

Contraction and valves & systemic resistance coupling

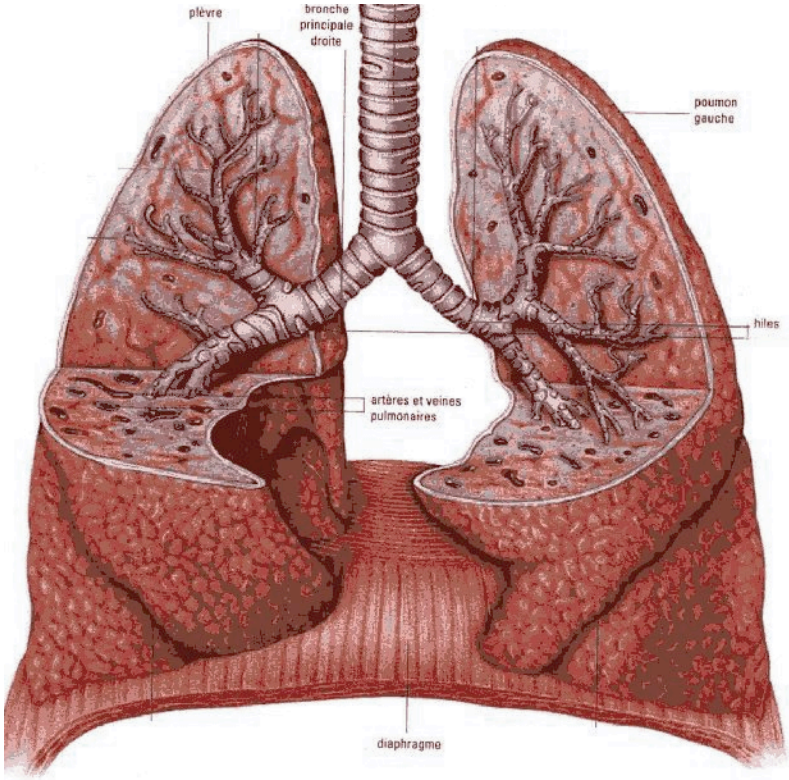
Aortic pressure

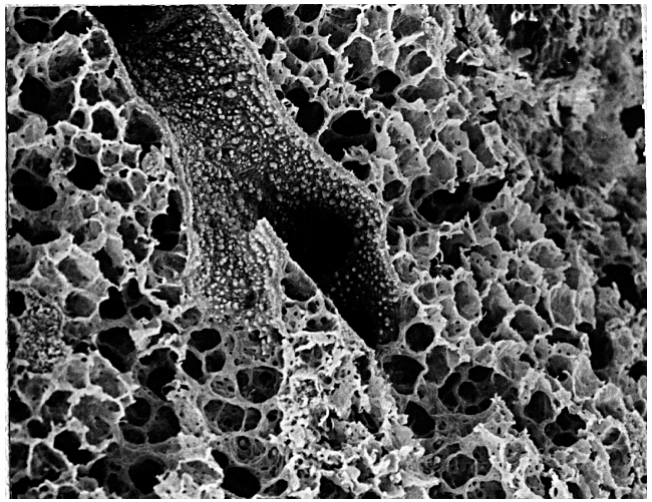
Coronary circulation

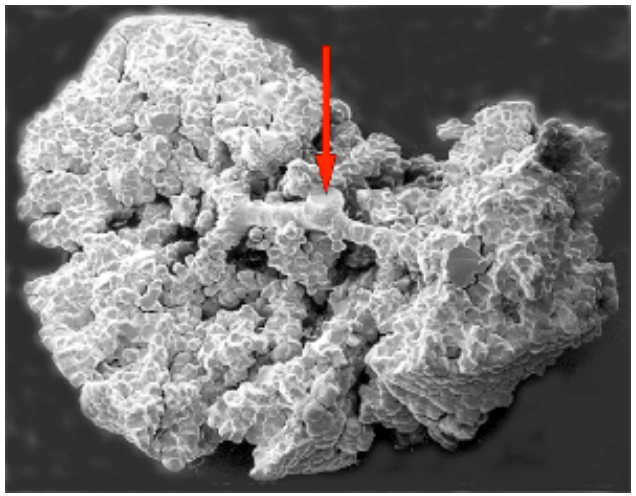




The respiratory system



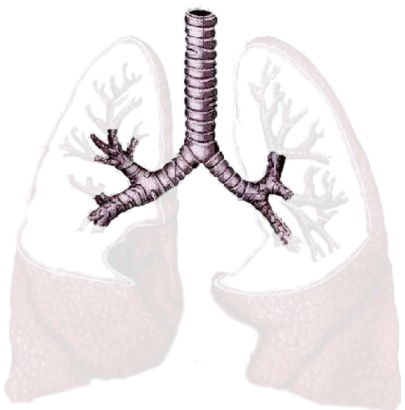


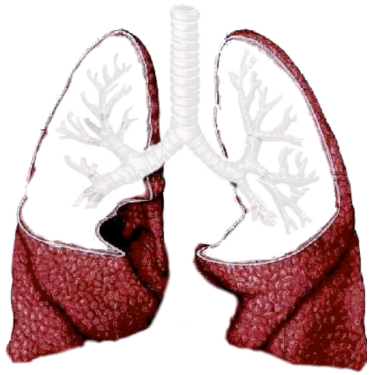


Multiscale Modeling

L. Baffico (Univ. Caen), C. G. (INRIA),
B. Maury (Univ. Orsay), A. Soualah (Univ. Orsay)

$$\Pi_i - P_i = R_i \int_{\Gamma_i} \mathbf{u}\cdot\mathbf{n}$$

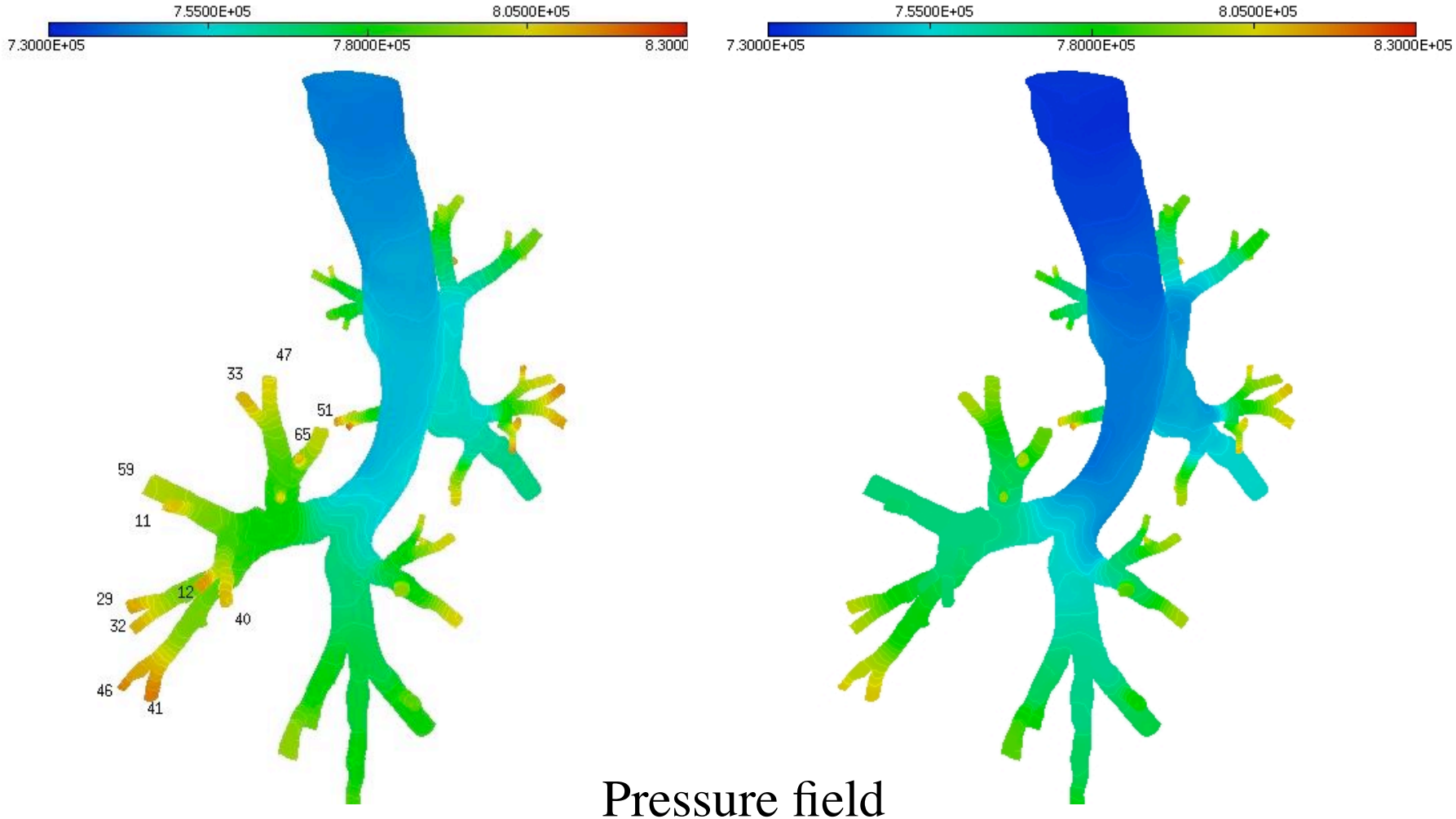




$$\begin{aligned}
\rho\frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u}\cdot\nabla)\mathbf{u} - \mu\Delta\mathbf{u} + \nabla p &= 0, && \text{in } \Omega \\
\nabla\cdot\mathbf{u} &= 0, && \text{in } \Omega \\
\mu\nabla\mathbf{u}\cdot\mathbf{n} - p\mathbf{n} &= -\Pi_i, && \text{on } \Gamma_i
\end{aligned}$$

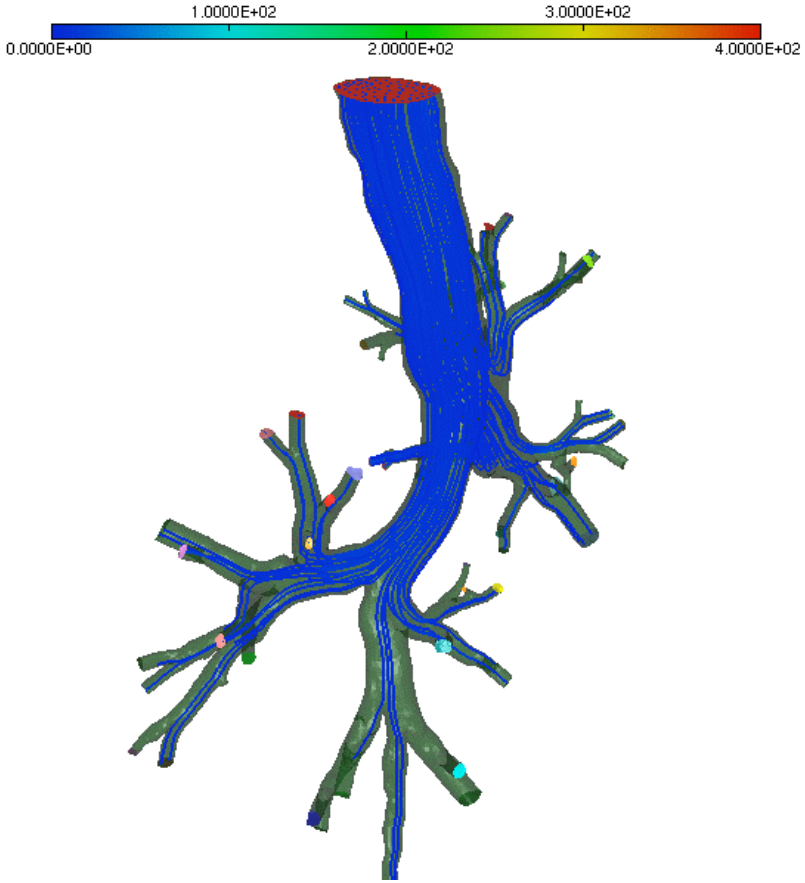
$$m\frac{d}{dt}\int_{\Gamma_0}\mathbf{u}\cdot\mathbf{n} + k\int_0^t\left(\int_{\Gamma_0}\mathbf{u}\cdot\mathbf{n}\right) = -kQ_0 + f_{ext} + P$$

Numerical simulations

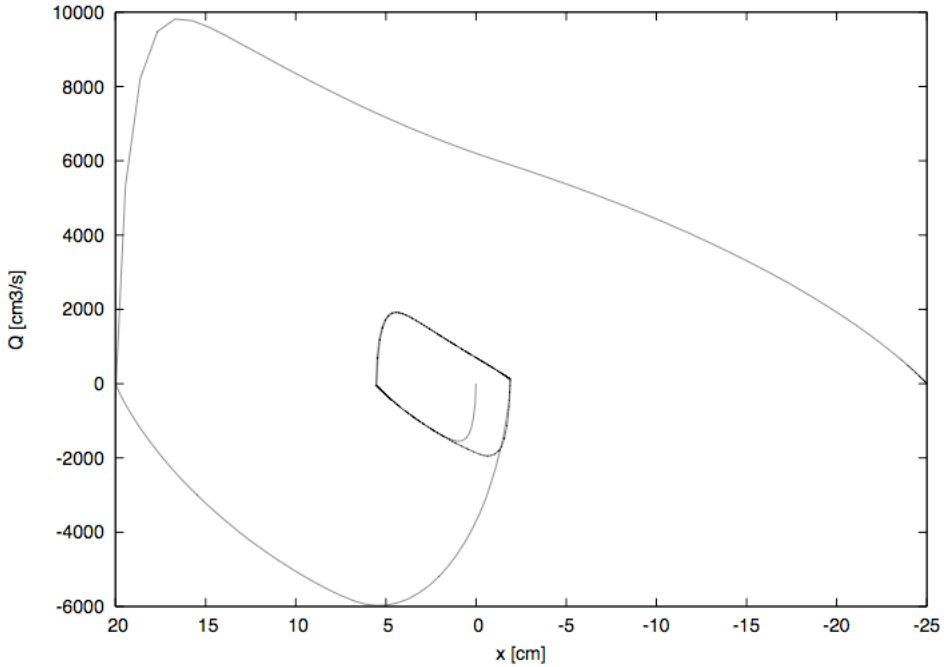


Pressure field

Numerical simulations



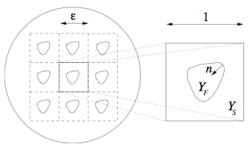
Streamlines

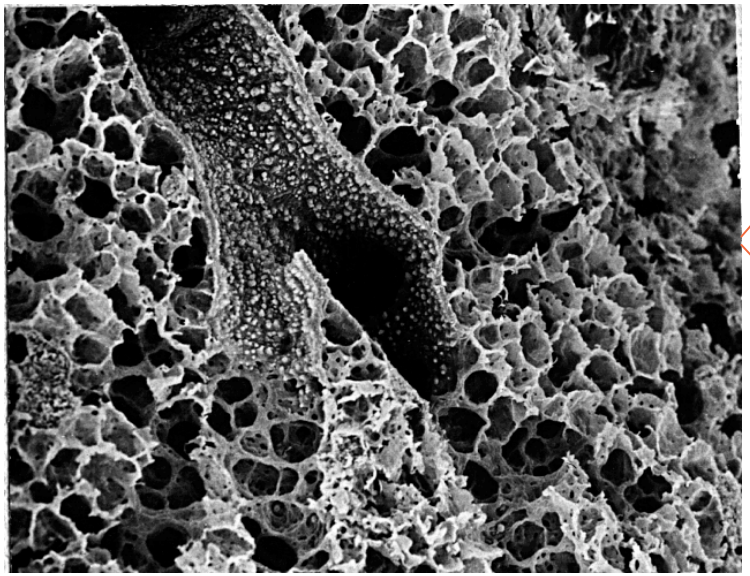


Volume- Flux curve

Multiscale Modeling

- Modelling of the parenchyma tissue





Homogenization

L. Baffico, C. G., Y. Maday, A. Osses

$$
\begin{aligned}
-\mathrm{div}\,\sigma(d^\varepsilon) &= f_S, \quad \text{in } \Omega_\varepsilon,\\
\sigma(\mathbf{d}^\varepsilon)\mathbf{n}_\varepsilon + \frac{A}{\varepsilon^N}\left(\int_{\Gamma_\varepsilon^k} \mathbf{d}_\varepsilon \cdot \mathbf{n}_\varepsilon \, d\gamma\right)\mathbf{n}_\varepsilon &= 0, \quad \text{on } \Gamma_\varepsilon^k,\ \forall k \in Z_\varepsilon^k\\
\sigma(d^\varepsilon)n &= 0, \quad \text{on } \Gamma_N,\\
d^\varepsilon &= 0, \quad \text{on } \Gamma_D.
\end{aligned}
$$

Effect of the bronchial tree

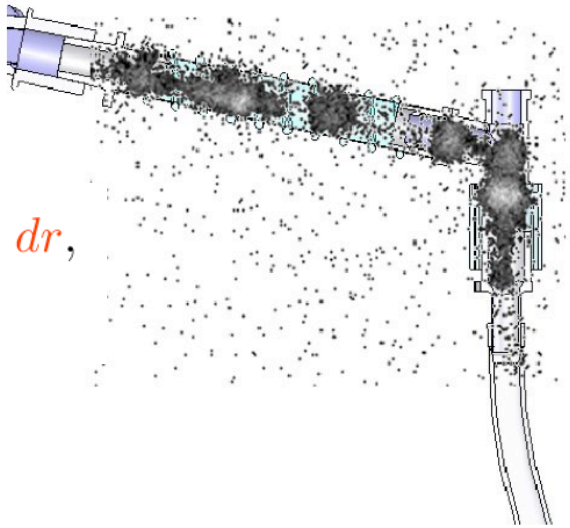
C. G., B. Maury, N. Meunier

$$
\rho\,\alpha_s\,\partial_{tt}u(x,t) - k\,\partial_{xx}u(x,t) - \partial_x \int_0^1 K(x,y)\,\partial_{ty}u(y,t)dy = \rho\,\alpha_s f(x,t),
$$

Multiphysics Modeling

- Fluid-Particle Interaction

L. Boudin, L. Desvillettes, C. G., A. Moussa, M. Thiriet, L. Weynans



- Navier-Stokes equations

$$\begin{cases} \partial_t u + \nabla_x \cdot (u \otimes u) + \nabla_x p = -\displaystyle\int_{v,r} m \frac{u-v}{\tau} \, f \, dv \, dr, \\ \nabla_x \cdot u = 0. \end{cases}$$

- Kinetic equation

$$\partial_t f + \nabla_x \cdot (f v) + \nabla_v \cdot \left(f \frac{u-v}{\tau} \right) + \partial_r (f \chi) = Q_{\mathrm{bup}}(f).$$

Inserm Tours

Conclusions

- Mathematical Modeling: Multiphysics/multiscale modeling
- Mathematical Analysis: well posedness, long time behavior, controlabillity…
- Design of efficient schemes, numerical analysis,
- 3D Simulations,
- Comparision with experimental/medical data: inverse problems.