Development of numerical model for the fluid-structure interaction problem in human body

*1. Department of Mechanics Engineering, The University of Tokyo

*2. Computational Science Research Program, Organ and Body Scale Team, RIKEN

Satoshi II[*1], Kazuyasu Sugiyama[*1], Shintaro Takeuchi[*1],

Shu Takagi[*1,*2] and Yoichiro Matsumoto[*1]

Young Investigators Symposium 2008

Background

Organ and human body simulation

The fluid-structure interaction problem is quite important in human circulation system such as blood flow and vessel wall, pulmonary respiration, etc.

Data format

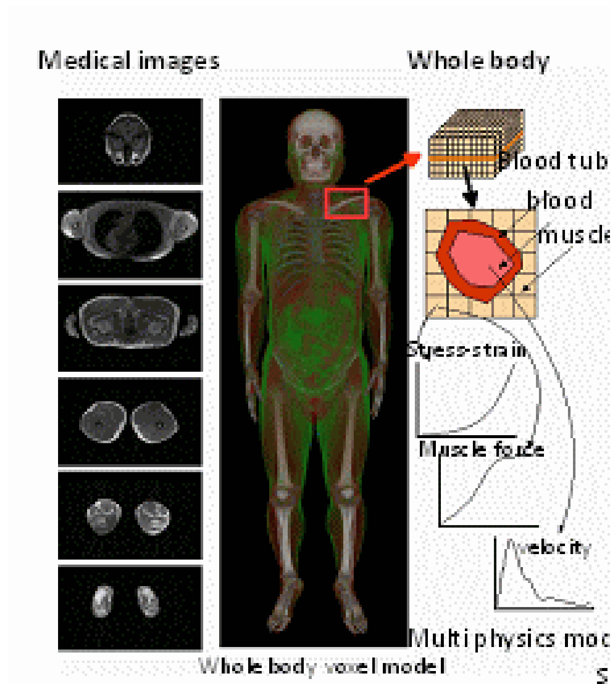
CT, MRI,
Ultrasound Imaging Equipment

⇩

VOXEL data (Brilliance etc.)

⇩

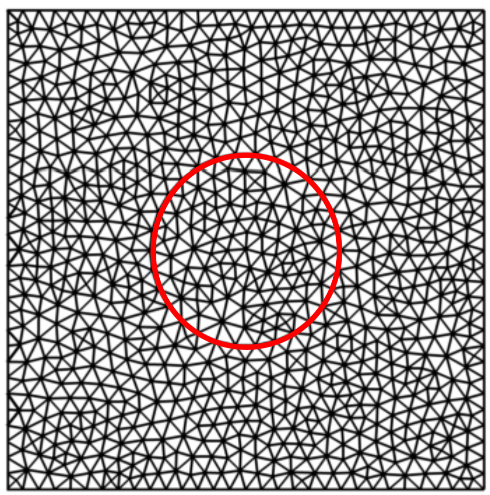
VOF data: a kind of volume data to represent geometry in Eulerian field



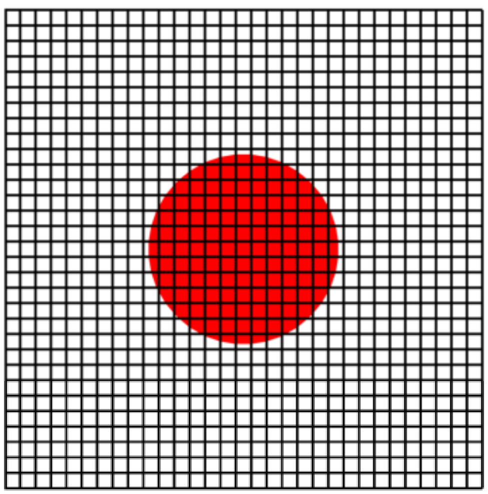
http://www.fel.t.u-tokyo.ac.jp/~takagi/zouki/a.html

Objective

Develop the numerical model for the fluid-structure interaction problem based on Eulerian framework, using VOF information.



Lagrangian framework



Eulerian framework with VOF function

Current fluid-structure interaction model

*Private communication (2008).

$$\nabla \cdot \mathbf{v} = 0, \quad \rho \frac{\mathrm{D}\mathbf{v}}{\mathrm{D}t} = -\nabla p + \nabla \cdot \boldsymbol{\sigma}, \quad \frac{\mathrm{D}\mathbf{B}}{\mathrm{D}t} = \mathbf{L} \cdot \mathbf{B} + \mathbf{B} \cdot \mathbf{L}^{\mathrm{T}}, \quad \frac{\mathrm{D}\phi}{\mathrm{D}t} = 0$$

- Averaged density

$$\rho = \phi^{(f)}\rho^{(f)} + \phi^{(s)}\rho^{(s)}$$

- Averaged shear stress

$$\boldsymbol{\sigma} = \phi^{(f)}\boldsymbol{\sigma}^{(f)} + \phi^{(s)}\boldsymbol{\sigma}^{(s)}$$

ρ: Density	$\mathbf{B}$: Left Cauchy-Green Deformation tensor
$\mathbf{v}$: Velocity vector	$\mathbf{L}$: Velocity gradient tensor
p: Pressure	ϕ: Volume of fraction/fluid
$\boldsymbol{\sigma}$: Shear stress tensor	

- Constitute equation

$$\boldsymbol{\sigma}^{(f)} = 2\mu\mathbf{D} \quad \text{(Newtonian fluid)} \qquad \mathbf{D} = \frac{1}{2}(\mathbf{L} + \mathbf{L}^T)$$

$$\left(\begin{array}{l} \boldsymbol{\sigma}^{(s)} = G(\mathbf{B}\cdot\mathbf{B} - \mathbf{B}) \quad \text{(St.Venant-Kirchhoff material),} \\ \qquad \text{or} \\ \boldsymbol{\sigma}^{(s)} = G(\mathbf{B} - \mathbf{I}) \quad \text{(Neo-Hookean material).} \end{array} \right.$$

Numerical algorithm

Fractional-step method

(1) VOF advection

$$\frac{\mathrm{D}\phi^n}{\mathrm{D}t} = 0$$

(2) B calculation

$$\frac{\mathrm{D}\mathbf{B}^n}{\mathrm{D}t} = \mathbf{L}^n \cdot \mathbf{B}^n + \mathbf{B}^n \cdot \mathbf{L}^{n\mathrm{T}}$$

(3) Advection & shear stress

$$\rho\frac{\mathrm{D}\mathbf{v}^n}{\mathrm{D}t} = \nabla \cdot \boldsymbol{\sigma}^n$$

(4) Pressure poisson

$$\nabla \cdot \left(\frac{\nabla p^{n+1}}{\overline{\rho}}\right) = \frac{\nabla \cdot \mathbf{v}^*}{\Delta t}$$

(5) Velocity projection

$$\frac{\mathbf{v}^{n+1} - \mathbf{v}^*}{\Delta t} = -\nabla p^{n+1}$$

Discretization

Temporal discretization

・1st-order forward Euler method

Spatial discretization

・VOF advection: THINC/WLIC method [1],[2]

・Advection: 3rd-order upwind FV method

・Non-advection, poisson, projection: 2nd-order central method

[1] F. Xiao, Y. Honma, T. Kono, Int. J. Numer. Meth. Fluids, 48 (2005) 1023-1040.

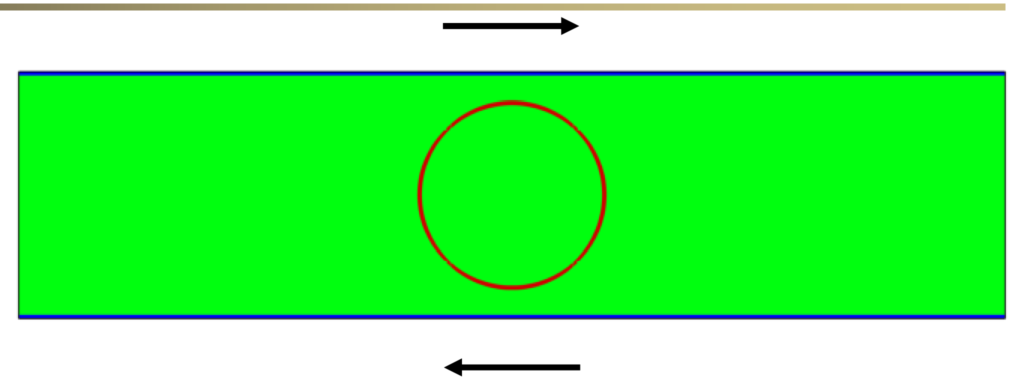
[2] K. Yokoi, J. Comput. Phys., 226 (2007) 1985-2002.

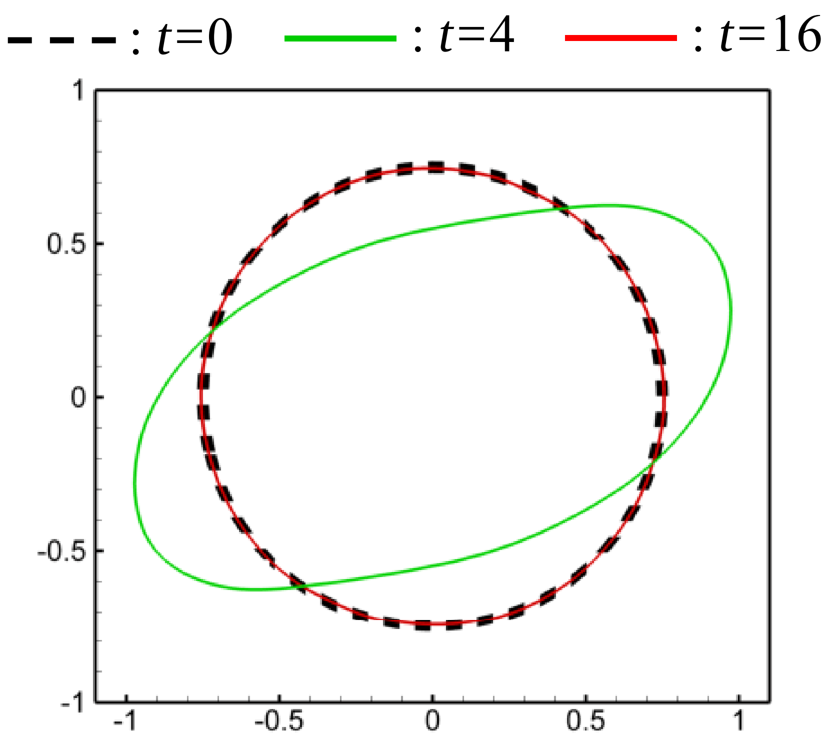
Circular material under shear flow

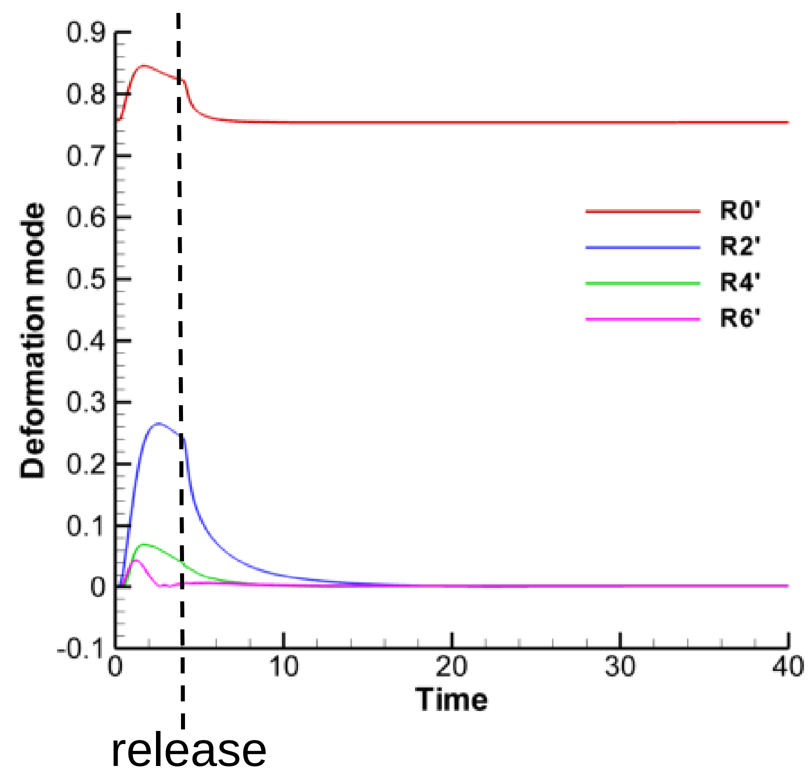
St. Venant-Kirchhoff model

$\rho^{(f)} = \rho^{(s)} = 1,\ \mu = 1,\ G = 4$

$N_x \times N_y = 256 \times 64$





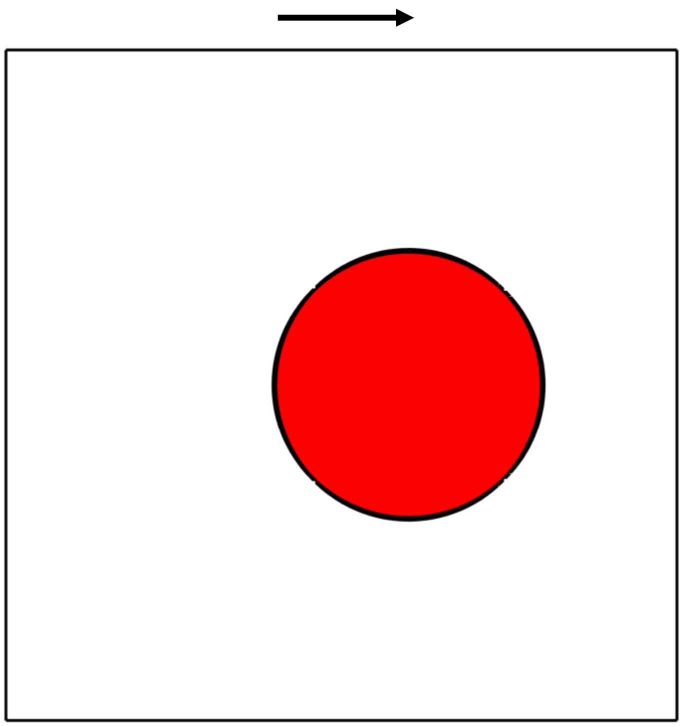


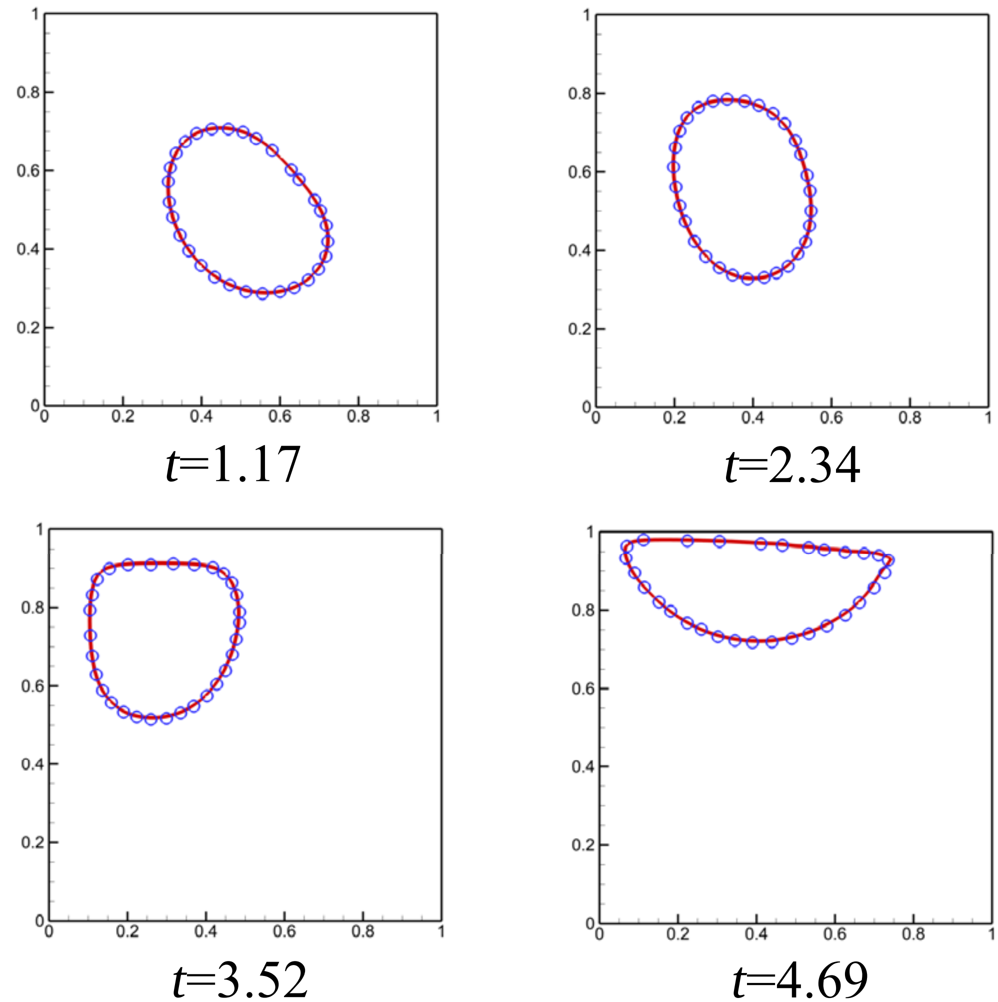
Lid-driven cavity flow with a circular material

Neo-Hookean model

$\rho^{(f)} = \rho^{(s)} = 1$, $\mu = 0.01$, $G = 0.1$

$Nx \times Ny = 128 \times 128$

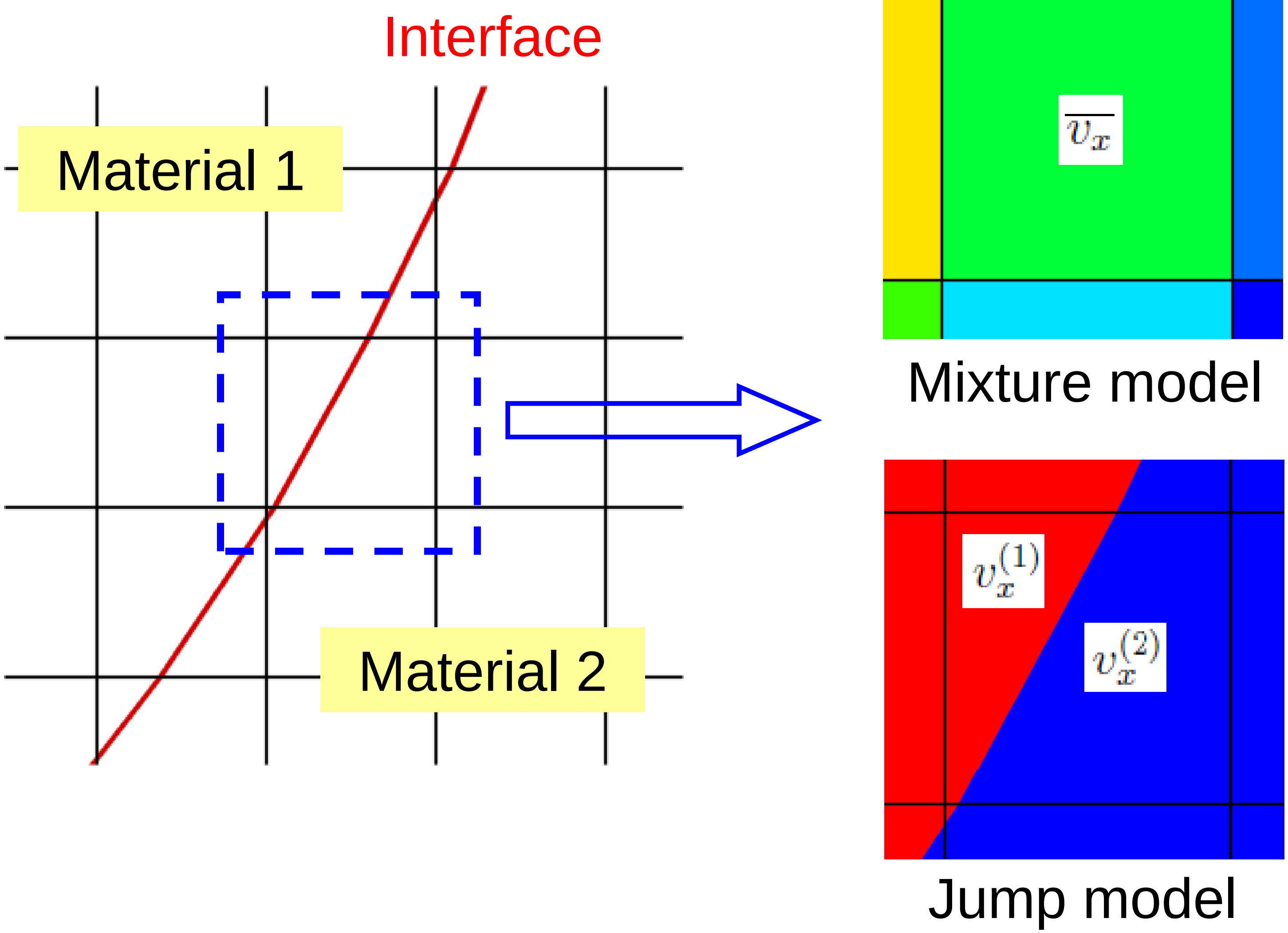




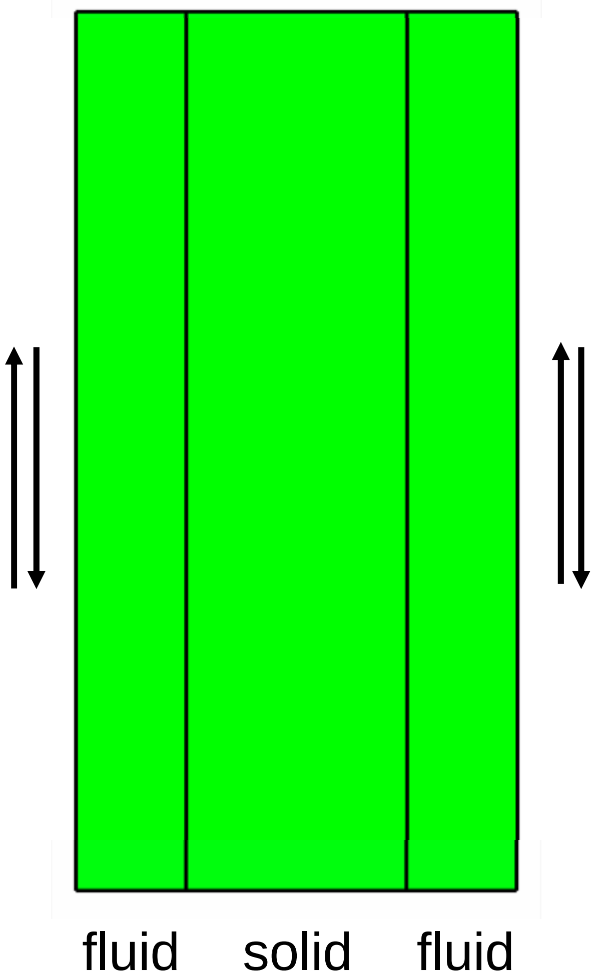
– Present ○ Zhao (Lagrangian model)*

* H. Zhao, J.B. Freund, R.D. Moser, J. Comput. Phys., 227 (2008) 3114-3140.

High order treatment for the interface element



1D linear fluid-elastic model (parallel fluid-structure layers)



u: Displacement v: Velocity

$$\partial_t v = \sigma_x$$

$$\partial_t (u_x) = v_x$$

$$\sigma = \begin{cases} \mu v_x & \text{fluid} \\ G u_x & \text{solid} \end{cases}$$

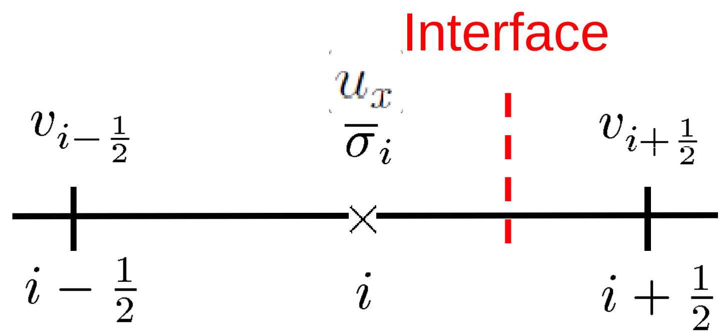
$$v_x \equiv \frac{\partial v}{\partial x}$$

Discretization

Time evolution equation

$$\frac{\mathrm{d}v_{i-\frac{1}{2}}}{\mathrm{d}t} = \frac{1}{\Delta x}\left(\overline{\sigma}_i - \overline{\sigma}_{i-1}\right)$$

$$\frac{\mathrm{d}u_{xi}^{(s)}}{\mathrm{d}t} = v_{xi}^{(s)}$$



Cell-averaged stress

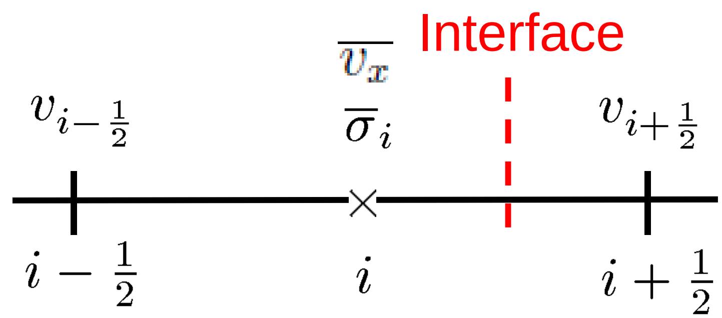
$$\overline{\sigma}_i = \phi_i^{(f)}\sigma_i^{(f)} + \phi_i^{(s)}\sigma_i^{(s)} = \phi_i^{(f)}\mu v_x^{(f)} + \phi_i^{(s)} G u_x^{(s)}$$

(ϕ: VOF function)

Interface model

Mixture model

$$\begin{cases} v_{xi}^{(f)} = \overline{v_{xi}} \\ v_{xi}^{(s)} = \overline{v_{xi}} \end{cases}$$



Jump model*

*Private communication (2008).

$$\begin{cases} v_{xi}^{(f)} = \overline{v_{xi}} + m_i^{(f)} \\ v_{xi}^{(s)} = \overline{v_{xi}} + m_i^{(s)} \end{cases}$$

$$\begin{cases} m_i^{(f)} = \phi^{(s)} \dfrac{-\mu\overline{v_x} + G(u_x^n + \Delta t\overline{v_x})}{\phi^{(s)}\mu + \phi^{(f)}\Delta tG} \\ m_i^{(s)} = -\phi^{(f)} \dfrac{-\mu\overline{v_x} + G(u_x^n + \Delta t\overline{v_x})}{\phi^{(s)}\mu + \phi^{(f)}\Delta tG} \end{cases}$$

Jump condition

$$[v] = v^{(f)} - v^{(s)} = 0$$

$$[\sigma] = \sigma^{(f)} - \sigma^{(s)} = \mu v_x^{(f)} - Gu_x^{(s)} = 0$$

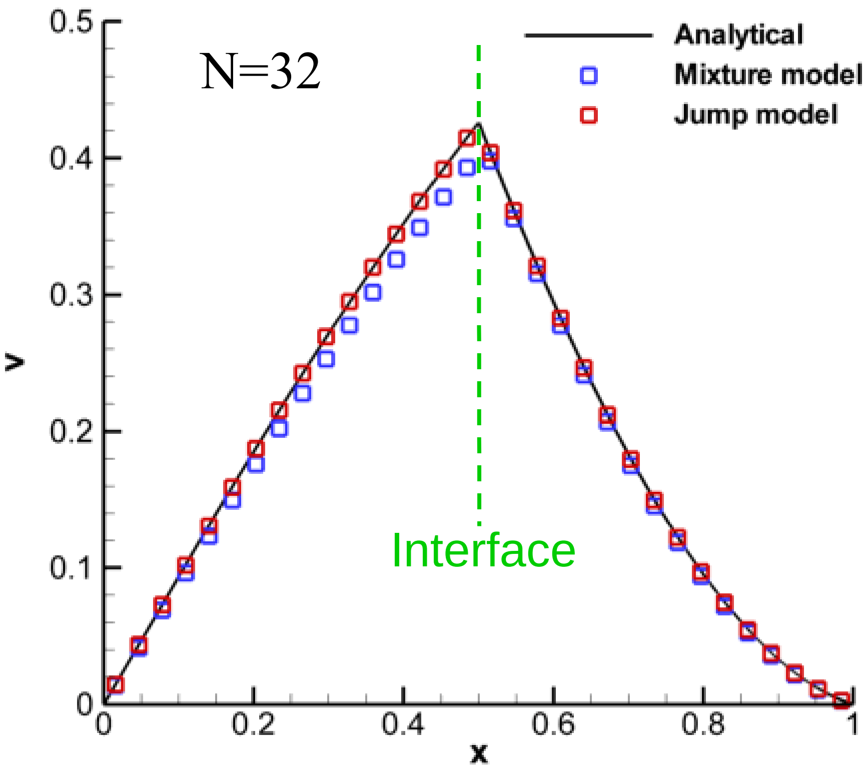
$$\overline{v_{xi}} = \frac{v_{i+\frac{1}{2}} - v_{i-\frac{1}{2}}}{\Delta x}$$

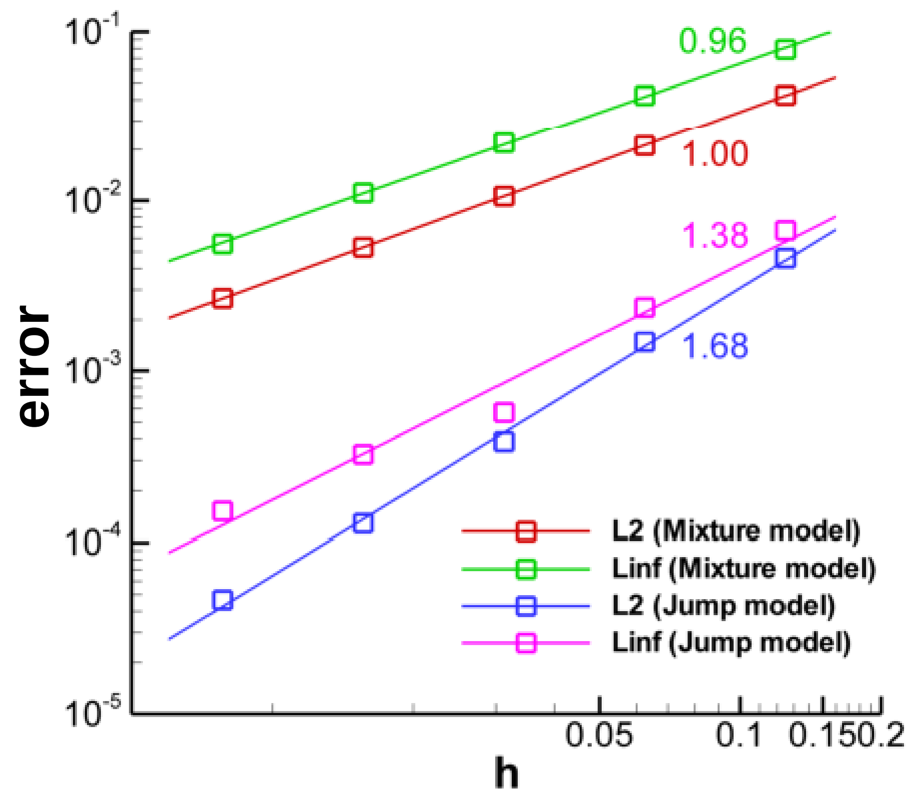
Example: 1D linear fluid-elastic model

$$\begin{cases} \Omega|_{\text{solid}} & (0 \le x \le 0.5), \\ \Omega|_{\text{fluid}} & (0.5 < x \le 1). \end{cases}$$

$\rho^{(f)} = \rho^{(s)} = 1$, $\mu = 1$, $G = 5$

N=8, 16, 32, 64, 128





Numerical results and convergence rate at t=2

Summary

- Simple fluid-structure interaction model has been constructed in Eulerian framework.

  (VOF function, left Cauchy-Green deformation tensor, St. Venant-Kirchhoff material, linear hyperelastic material, mixture model)

- Adequate numerical model with jump conditions was obtained for 1D fluid-elastic interaction problem.

Future work

- Physical constitutive equation
- More accurate numerical model based on jump model for multi-dimensional fluid-structure interaction problem
- Comparison between numerical result and experimental result
- Real-case problem