Properties of Hot Strongly Interacting Matter from Large Scale Numerical Simulations

Péter Petreczky
Physics Department

What Quantum Chromo-Dynamics (QCD) tells about the properties of strongly interacting matter at very high temperatures when solved numerically in discretized space-time => Lattice QCD (LQCD)

NSAC: one of the 5 questions that “continue to drive nuclear science in the coming decades” is “What are the properties of hot nuclear matter?”

National Research Council report: “Connecting Quarks with Cosmos: Eleven Science Questions for a New Century” listed “Are there new states of matter at exceedingly high density and temperature?”

Deconfinement at high temperature and density

Hadron Gas

Quark Gluon Plasma (QGP)

Transition

Why this is interesting:

Basic properties of strong interactions, interpretation of heavy ion experiments

Astrophysics: compact stars

Cosmology: neutrino and WIMP dark matter

LQCD

Quark Gluon Plasma (QGP)
Relativistic Heavy Ion Collisions
\[ \text{Au} + \text{Au} \quad \sqrt{s} = 7 - 200\text{GeV} \]
\[ \epsilon_{\text{max}} \simeq 15 - 30\text{GeV/fm}^3 \]
\[ \epsilon_{\text{min}} \simeq 0.1 - 0.6\text{GeV/fm}^3 \]

$2B$ investment of DOE

Monte-Carlo Simulations of Lattice QCD at $T > 0$

IBM BG/L @ BNL (NYBlue) and LLNL:

Algorithm:
Rational Hybrid Monte-Carlo (RHMC)
(x20 speedup for physical quark masses)

SciDAC software:
Columbia Physics System (CPS),
MIMD Lattice Collaboration Code (MILC)

http://usqcd.jlab.org/usqcd-software/
Lattice QCD at T>0 and RHIC

- Bulk particle spectra
- Heavy quark bound states
- Thermal photons and dileptons

Transition temperature, equation of state, susceptibilities

Spatial and temporal correlation functions, spectral function, transport coefficients
Finite Temperature QCD and its Lattice Formulation

\[ \langle O \rangle = \text{Tr} O e^{-\beta H - \mu N} \]

\[ \beta = \frac{1}{T} \]

Integral over functions

Lattice

Integral with very large (but finite) dimension ( > 1000)

Continuum limit

\[ N_\tau \to \infty, \quad N_\sigma / N_\tau \text{ fixed} \]

Costs:

\[ \sim a^{-7} \sim N_\tau^7 \]

Improved discretization schemes are needed: p4, asqtad, stout, HISQ
Deconfinement: entropy, pressure and energy density

- Free gas of quarks and gluons: 18 quark + 18 anti-quarks + 16 gluons
- Meson gas: 3 light d.o.f.

- Meson gas = 3 light d.o.f.
- Free gas of quarks and gluons = 18 quark + 18 anti-quarks + 16 gluons
  = 52 mass-less d.o.f.

- Rapid change in the number of degrees of freedom at $T = 170-200\text{MeV}$: deconfinement
- Deviation from ideal gas limit is about 10% at high $T$ consistent with the perturbative result
- No large discretization errors in the pressure and energy density at high $T$
- No continuum limit yet!
Deconfinement and color screening

Free energy of static quark anti-quark pair shows Debye screening at high temperatures

\[ F_1(r) = -\frac{4\alpha_s}{3} \exp(-m_D r) + 2F_Q(T), \quad m_D \sim T \]

\[ r_{\text{bound}} > \frac{1}{m_D} \]

Melting of bound states of heavy quarks => quarkonium suppression at RHIC

HISQ, \( N_s=8 \)
HISQ, \( N_s=6 \)
p4, \( N_s=8 \)
HIS, extr.

Polyakov loop

\[ L_{\text{ren}} = \exp(-F_Q(T)/T) \]

Free energy of a static quark

Large in confined phase \( \sim 600 \text{MeV} \)
Decreases in the deconfined phase

\[ F_Q(T') \approx \Lambda_{QCD} - C_F\alpha_s m_D \]
Chiral symmetry of QCD in the vacuum and for $T>0$

- **Chiral symmetry**: For light quarks $m_u, d \ll \Lambda$ and QCD Lagrangian

  $$SU_A(2) \text{ symmetry } \psi \rightarrow e^{i\phi T^a \gamma_5} \psi$$
  $$\psi_{L,R} \rightarrow e^{i\phi_{L,R} T^a} \psi_{L,R}$$

  The vacuum (ground state) is not spontaneous symmetry breaking or Nambu-Goldstone realization of the symmetry.

  Hadrons with opposite parity have very different masses.

  Chiral symmetry is expected to get restored at high $T$: \[ \langle \bar{\psi} \psi \rangle = 0 \]

- For vanishing $u,d$-quark masses the chiral transition is either 1$^{\text{st}}$ order or 2$^{\text{nd}}$ order phase transition.

- For physical quark masses there could be a 1$^{\text{st}}$ order phase transition or crossover.

  Evidence for 2$^{\text{nd}}$ order transition in the chiral limit $\Rightarrow$ universal properties of QCD transition:

  $$SU_A(2) \sim O(4)$$
  relation to spin models.
Chiral symmetry restoring transition

Subtracted chiral condensate:

\[ \Delta_{s,l}(T) = \frac{\langle \bar{\psi}\psi \rangle_T - \frac{m_l}{m_s} \langle \bar{s}s \rangle_T}{\langle \bar{\psi}\psi \rangle_{T=0} - \frac{m_l}{m_s} \langle \bar{s}s \rangle_{T=0}} \]

\[ \frac{\partial \Delta_{s,l}(T)}{\partial T} \bigg|_{T=T_\Delta} \sim m_l^{(\beta-1)/(\beta\delta)} \]

Disconnected chiral susceptibility:

\[ \frac{\chi_{\text{disc}}(T)}{T^2} = \langle (\bar{\psi}\psi)^2 \rangle - \langle \bar{\psi}\psi \rangle^2 \]

\[ \chi_{\text{disc}}(T = T_\chi) \sim m_l^{1/\delta-1} \]

\[ T_\chi = T_\Delta \]

for \( m_l \to 0 \)

HISQ, \( N_t=8 \):

\[ T_\Delta = (165 \pm 3 \pm 5(\text{scale}))\text{MeV} \]

\[ T_\chi = (164 \pm 4 \pm 5(\text{scale}))\text{MeV} \]
Chiral transition and universal scaling

For sufficiently heavy strange quark and small light quark masses the chiral transition is governed by universal O(4) (O(2)) scaling:

\[ f(T, m_l, m_s) = f_{\text{reg}}(T, m_l, m_s) + f_s(t, h), \quad t = \frac{1}{t_0} \frac{T}{T_c}, \quad H = m_l/m_s, \quad h = H/h_0 \]

\[ M \leftrightarrow \langle \overline{\psi} \psi \rangle \]

\[ M = -\frac{\partial f_s(t, h)}{\partial H} = h^{1/\delta} f_G(z), \quad z = t/h^{1/\beta \delta} \]

\[ f_\chi = m_s^2 \chi \frac{h_0^{1+1/\delta}}{T^4} (m_l/m_s)^{1-1/\delta} \]

\[ p4, N_t = 4 \]

Ejiri et al, PRD 80 (09) 094505
QCD thermodynamics at non-zero chemical potential

Taylor expansion:

\[
\frac{p(T, \mu_B, \mu_S, \mu_I)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!l!} \chi_{ijl}^{BSI} \cdot \mu_i^B \cdot \mu_j^S \cdot \mu_k^I
\]

hadronic

\[
\frac{p(T, \mu_u, \mu_d, \mu_s)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijl}^{udS} \cdot \mu_i^u \cdot \mu_j^d \cdot \mu_k^S
\]

quark

\[
\chi_{abc}^{i,j,k} = \frac{\partial^i}{\partial \mu_a^i} \frac{\partial^j}{\partial \mu_b^j} \frac{\partial^k}{\partial \mu_c^k} \frac{1}{VT^3} \ln Z(T, V, \mu_a, \mu_b, \mu_c)|_{\mu_a=\mu_b=\mu_c=0}
\]

Taylor expansion coefficients give the fluctuations and correlations of conserved charges, e.g.

\[
\frac{\chi_X^2}{T^2} = \frac{\chi_X}{T^2} = \frac{1}{VT^3} (\langle X^2 \rangle - \langle X \rangle^2)
\]

\[
\frac{\chi_{11}^{XY}}{T^2} = \frac{1}{VT^3} (\langle XY \rangle - \langle X \rangle \langle Y \rangle)
\]
Deconfinement: fluctuations of conserved charges

\[
\frac{\chi_B^{SB}}{T^2} = \frac{1}{VT^3} (\langle B^2 \rangle - \langle B \rangle^2)
\]
baryon number

\[
\frac{\chi_I^{SB}}{T^2} = \frac{1}{VT^3} (\langle I^2 \rangle - \langle I \rangle^2)
\]
isospin

\[
\frac{\chi_S^{SB}}{T^2} = \frac{1}{VT^3} (\langle S^2 \rangle - \langle S \rangle^2)
\]
strange quark number (strangeness)

Ideal gas of quarks:

\[
\chi_B^{SB} = \frac{T^2}{3}, \quad \chi_I^{SB} = \frac{T^2}{2}
\]

\[
\chi_S^{SB} = T^2
\]
conserved charges carried by light quarks


Conserved charges are carried by massive hadrons
At sufficiently high $T$ fluctuations can be described by perturbation theory because of asymptotic freedom.

The quark number susceptibilities for $T > 300\text{MeV}$ agree with resummed perturbative predictions.

Blaizot et al, PLB 523 (01) 143

hadrons are the relevant d.o.f. at low $T$
$\Rightarrow$ hadron gas + interactions
(approximated by s-channel resonances)
$\Rightarrow$ non-interacting hadron resonance gas (HRG)

Reasonable agreement between lattice results and HRG the remaining discrepancies are due to the lack of continuum extrapolation
Correlations of conserved charges

- Correlations between strange and light quarks at low $T$ are due to the fact that strange hadrons contain both strange and light quarks but very small at high $T$ (>250 MeV) => weakly interacting quark gas

- For baryon-strangeness correlations HISQ results are close to the physical HRG result, at $T>250$ MeV these correlations are very close to the ideal gas value

- The transition region where degrees of freedom change from hadronic to quark-like is broad ~ 50 MeV

Critical end-point and isentropic equation of state

If all expansion coefficients are positive there is a singularity for real $\mu_B$. The largest temperature for which all expansion coefficients are positive $T_n$ provides an estimate for

$$T^\text{CEP} = T_n, \quad n \to \infty$$

Radius of convergence at $T \sim T^\text{CEP}$ provides an estimate for $\mu_B^\text{CEP}$

$$\rho_n = \sqrt{c_{n+2}/c_n}, \quad \mu_B^\text{CEP} = \rho_n, \quad n \to \infty$$

Using Taylor expansion one can calculate the entropy density at finite $\mu_B$ and the set of $(T, \mu_B)$ which corresponds to constant ratio of entropy to baryon number.
Summary

Lattice QCD show that at high temperatures strongly interacting matter undergoes a transition to a new state QGP characterized by deconfinement and chiral symmetry restoration.

Thanks to significant increase in computational resources and SciDAC software for the first time it is possible to calculate thermodynamic quantities in QCD with controlled systematic errors.

We see evidence that the relevant degrees of freedom are quarks and gluons; lattice results agree well with perturbative calculations, while at low $T$ thermodynamics can be understood in terms of hadron resonance gas. The deconfinement transition can be understood as transition from hadron resonance gas to quark gluon gas, it is gradual and analogous to ionized gas – plasma transition (implications for sQGP and early thermalization at RHIC?)

Different calculations improved staggered actions are like agree in the continuum limit resulting in a chiral transition temperature (155 -165) MeV.

Open challenges for Exaflop era: phase diagram at finite baryon density, calculation of dynamical structure functions of QGP.
Early Universe
Decoupling of Weakly Interacting Massive Particles (WIMP) a popular dark matter candidate
Hindmarsh, Philipsen, PRD 71 (05) 087302
Production of light sterile neutrino (WDM)
M. Laine, PoS LAT2006 (06) 014

Spectrum of primordial gravitational waves
Watanabe, Komatsu, PRD 73 (06) 123515
enhancement of strangeness fluctuations compared to the previous calculations on $N_T=8$ with lattice due to reduced discretization errors

All the lattice results are below the HRG. Hadron masses on the lattice are large than physical due to $a^2$ discretization effects this needs to be taken into account =>

**good agreement between lattice and modified HRG**, Huovinen, P.P, arXiv:0912.2541
Langelage, Philipsen arXiv:1002.1507

lattice calculations with stout and HISQ action are expected to give consistent results in the continuum limit and agree with physical HRG

stout continuum : arXiv:1005.3508